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Genralisation And Some Characteristics Of Tri-bonacci Sequence

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Abstract— The Fibonacci sequence is one of the most intriguing number sequences, and it continues to provide ample opportunities for professional and amateur mathematicians to make conjecture and to expend the mathematical horizon.
The sequence is so important that an organization of mathematicians, the Fibonacci association, has been formed for the study of Fibonacci and related integer sequences. The association was founded in 1963 by Verner E.Hoggatt; jr (1912-1980) of San Jose state College (now San Jose state university), California, and Brother publishes The Fibonacci quarterly, devoted to articles related to integer sequences.
A close look at the Fibonacci sequence reveals that it has a fascinating property: In the number of Fibonacci and Lucas numbers, every element, expect for the first two, can be obtained by adding its two immediate predecessors.
Now, suppose we are given three initial condition and add the three immediate predecessors to compute their successor in a number sequence. Such a sequence is the tribonacci sequence, originally studied in 1963 by M. Feinberg when he was a 14-year-old ninth grader at Susquehanna Township Junior High School in Pennsylvania (1963a).

Index Terms— Linear recurrent sequence; decomposition of unlimited terms; tribonacci.

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1 INTRODUCTION

This work falls within the framework of number theory. We took as model the number of Fibonacci which are:

, , , , , , , ,... generated by the linear recurrent relation

With initial conditions

We talked about some their characteristics, especially from generalized Fibonacci sequence

With initial conditions

And the link between

$$F_n \text{ and } G_n ...$$

Our contribution as the same plan, we relied on a new numbers

, , , , , , , , , , , , , , , of de-fined by by the linear recurrent relation

$$T_n = T_{n-1} + T_{n-2} + T_{n-3} \quad n \geq 3$$

With initial conditions

$$T_1 = T_2 = 1, \quad T_3 = 2.$$

Similarly we worked to find the generalized of which is denoted by where

and

When we can proved that

more than as it has been identified by the theorem that every integer is able to written on the sum of distinct tribonacci numbers, with the help of special cases of which are (number of addition) and we were able to enrich this research by characteristics with promising prospects.

2 Fibonacci sequence and his generalized

We begin this research by display the Fibonacci sequence (F_n), Which will be a strong support of us in all our contribution in relation to the study some property of tribonacci sequence which we know a few about it by comparing with the previous sequence

2.1 Recursive definition of Fibonacci sequence

The following recursive relation define the n th Fibonacci number, F_n :

The eleven terms of Fibonacci are: 1 , 1 , 2 , 3 , 5 , 8 , 13 , 21 , 34 , 55 , 89 , ...

2.2 Generalized Fibonacci numbers

We consider the sequence $\{G_n\}$, where $G_1 = a$, $G_2 = b$ and $G_n = G_{n-1} + G_{n-2}$, $n \geq 3$. The first terms are : ,

, , , $2a+3b$, $3a+5b$, is called the generalized Fibonacci sequence (GFS).

We can remark that the coefficients of and in the various terms of this sequence. they follow an interesting pattern:

The coefficient of and are Fibonacci numbers. In fact, we can pinpoint these two Fibonacci coefficients as the following theorem [4] .

2.2.1 Theorem [5]

Let G_n denote the n th term of the (GFS). Then :

$$G_n = aF_{n-2} + bF_{n-1}$$

Proof.

By the principal of mathematical induction. Since

, the statement is true when $n = 3$.

Let k an arbitrary integer ≥ 3 . Assume that the statement is true for all integers , where $3 \leq i \leq k$. Then:

$$\begin{aligned} G_{k+1} &= G_k + G_{k-1} \\ &= (aF_{k-2} + bF_{k-1}) + (aF_{k-3} + bF_{k-2}) \\ &= a(F_{k-2} + F_{k-3}) + b(F_{k-1} + F_{k-2}) \\ &= aF_{k-1} + bF_k \end{aligned}$$

.

Thus, by the principle of mathematical induction (PMI) the formula holds for every integer .

Notice that this theorem is in fact true for all .

We study some properties in the following theorem.

2.2.2 Theorem

Let denote the n th term of the (GFS). Then :

$$\sum_{i=1}^{n} G_{k+i} = G_{n+k+2} - G_{k+2}$$

Proof

By precedent theorem

$$\begin{aligned} \sum_{i=1}^{n} G_{k+i} &= a\sum_{i=1}^{n} F_{k+i-2} + b\sum_{i=1}^{n} F_{k+i-1} \\ &= a(F_{n+k} - F_k) + b(F_{n+k+1} - F_{k+1}) \\ &= (aF_{n+k} + bF_{n+k+1}) - (aF_k + bF_{k+1}) \\ &= G_{n+k+2} - G_{k+2} \end{aligned}$$

3 Tribonacci sequence and his generalized

From here we begin our contributions in this research starting with the definition of tribonacci sequence passing to his generalized and characteristics, after we try to find the link between the tribonacci terms and the Fibonacci terms and generally between any integer

3.1Definition of tribonacci numbers

[5].

The tribonaci numbers T_n are defined by the recurrent linear relation

Where , and .

The first twenty tribonacci numbers are

, , , , , , , , , , , ,

, , , , , , ,

.

3.2 Generalized tribonacci sequence (GTS)

To this end, consider the sequence , where ,

, , and

The first twelve tribonacci numbers are: , , ,

, , ,

, , ,

, ,

.

This ensuing sequence is called **the generalized tribonacci sequence** (GTS).

Take a close look at the coefficients of and are tribonacci numbers, and the coefficient of is the sum of two tribonacci numbers. as the following theorem shows.

Let denote the th term of the GTS. Then T=Ta+ T+T b+Tc, % n5.

Proof By the principal of mathematics induction (PMI) since

The statement is true when

Assume the given statement is true for all integer , when

:

Then:

$$T'_{n+1} = T'_n + T'_{n-1} + T'_{n-2}$$

$$= T_{n-3}a + \left(T_{n-4} + T_{n-3}\right)b +$$
$$T_{n-2}c + T_{n-4}a + \left(T_{n-5} + T_{n-4}\right)b$$
$$+T_{n-3}c + T_{n-5}a + \left(T_{n-6} + T_{n-5}\right)b$$
$$+T_{n-4}c$$

$$= \left(T_{n-3} + T_{n-4} + T_{n-5}\right)a +$$
$$\left(\left(T_{n-4} + T_{n-5} + T_{n-6}\right) + \left(T_{n-3} + T_{n-4} + T_{n-5}\right)\right)b +$$
$$\left(T_{n-2} + T_{n-3} + T_{n-4}\right)c$$

Thus, by the principal of mathematics induction (PMI) the formula holds for every integer .

3.3 Definition

In the next we explore formula for the number of addition needed to compute recursively. For example, it takes two additions to compute ; that is .

3.4 Theorem

Let denote the number of additions needed to compute recursively. Then

$$a_n = T_{n-1} + T_{n-3} - 1$$

Where .

Proof

Since , the formula works when

Now, assume it is true for all positive integers , when

Then:

So

$$a_{k+1} = a_k + a_{k-1} + a_{k-2} + 2$$
$$= \left(T_{k-1} + T_{k-3} - 1\right) + \left(T_{k-2} + T_{k-4} - 1\right) +$$
$$\left(T_{k-3} + T_{k-5} - 1\right) + 2$$
$$= \left(T_{k-1} + T_{k-2} + T_{k-3}\right) + \left(T_{k-3} + T_{k-4} + T_{k-5}\right) - 1$$
$$= T_{k-1} + T_{k-3} - 1.$$

Thus by the strong version of mathematical induction, the formula holds for every .

It follows by the theorem that:

The statement is true when .

Let be an arbitrary integer . Assume the given statement is true for any integer , when

Then

We have

Then

Finally

Thus, by the principal of mathematics induction (PMI) the formula holds for every integer .

3.7 Theorem

For every integer we have:

Proof

By combining party to party and simplified we find

Then

Or

$$\sum_{i=2}^{n-2}\left(T_i + T_{i+1}\right) = T_n - T_4 = T_n - 4.$$

4. Theorem (representation of integers)

Let $\{T_n : n \in \}$ the sequence of triibonacci defined by , $T_3 = 2,$ and by recurrent relation , for any positive integer can be written as the sum of distinct tribonacci numbers.

Proof

Let an arbitrary positive integer, and such that . Put . If , we have finished, because . Else, Let with such that . If , we have finished, because in this case . Else we choose such that , consequently. This process is finished because the sequence of positive integer is croissant and so possibly we give for certain positive integer , in that case we have .

4.1 Remark

This representation is not unique.

5. Conclusion

The number theory is an important famous part of algebra in mathematics. It mainly concerned to the numbers and their characteristics and their kinds, uses in the numérotation and cryptology ...For more controling to all this the researchers went to dealing with numbers generated by mathematical formula generally. as odd, even and prime numbers, also by the terms of linear recurrent sequence with integer terms. The simple famous example which use in that art the Fibonacci sequence , ; after Lucas generalized Fibonaccci sequence depending on the coefficient of the two last terms as: , , ; such that two integers [3]. In 1930 Lehmer generalized Lucas sequence as: , , ; such that two integers [6] and so on. Than it come the tribonacci sequence in which to find the term we need to sum of three last terms beginning from , ; . Accordingly we exploit the same characteristics of and we find a generalized of tribonacci sequence which is denoted by and the link between and , . Also it enables us to prove that every integer can be writes in the form of sum of distinct tribonacci numbers.

Where fore can translate all characterestics of to similarly. Wherever we advanced in the study of this linear recurrent sequence we more control in the integers and whoever in the fascide applications like the cryptography and so on... More generalization can deal with it and recover meany results from it like our use of the sum of four terms before the term which we like calculate relief three and so on...

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